attempt was made to determine its location. However, the present computations do not model transition and the flow is assumed to be turbulent right at the leading edge of the airfoil. Additionally, the flow is incompressible and the Reynolds number is a little lower than that in the experiments. These reasons might explain some of the differences that can be noticed between the numerical and experimental results. Hysteresis in the aerodynamic coefficients for the computed solution can be observed for α lying between 17 and 19 deg. Qualitatively, these results look similar to those reported by Hoffman¹ for his experiments with the NACA 0015 airfoil for low FST.

The hysteretic behavior of the flow is caused by its ability to remember its past history. The starting point for the flows along the increasing angle branch is an attached flow, whereas it is a massively separated flow for the flows along the branch corresponding to the decreasing angle. The instantaneous vorticity and pressure fields for the two solutions for $\alpha = 18$ deg, in the fully developed state, are shown in Fig. 3. The solution along the increasing angle branch shows relatively low unsteadiness compared to the one that has been obtained with the decreasing angle of attack. This is also reflected in the time histories of the lift and drag coefficients for the two solutions (not shown here). The unsteady component of the aerodynamic coefficients for the flow with decreasing angle is significantly larger than that with the increasing angle. Because both of the computations have been carried out with identical finite element meshes, it can be concluded that it is the initial condition that is responsible for the multiplicity of solution.

Conclusion

Results have been presented for computation of flow past a NACA 0012 airfoil using RANS equations in conjunction with a Baldwin-Lomax turbulence model for closure. A well-proven stabilized finite element method that has been applied to various flow problems earlier has been utilized to solve the incompressible Navier-Stokes equations in the primitive variables formulation. Hysteresis in the flow has been observed for angles of attack close to the stall angles of the airfoil. The ability of the flow to remember its past history is responsible for its hysteretic behavior. For the same angle of attack, the flow obtained with the increasing angle results in an almost attached flow with higher lift and lower drag, whereas the one with decreasing angle of attack is associated with large unsteadiness, lower lift, and higher drag. These computations demonstrate that a simple turbulence model in conjunction with an accurate flow solver can replicate fairly complex physical phenomenon. The success of the turbulent flow calculations depend not only on the complexity of the turbulence model, but also on the accuracy of the underlying basic numerical scheme for solving RANS.

Acknowledgments

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J. Kallinderis Associate Editor

On Mixing Enhancement via Nozzle Trailing-Edge Modifications in High-Speed Jets

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Introduction

E XTENSIVE research has been conducted over the past few years on mixing enhancement using trailing-edge modifications in supersonic rectangularjets. The trailing-edge modifications (or cutouts) were either on the splitter plate in a half nozzle¹ or on the nozzle extension in a full nozzle.² The use of trailing-edge modifications of this type resulted in significant mixing enhancement in the underexpanded flow regime, moderate enhancement in the overexpanded regime, and no significant mixing enhancement in the ideally expanded flow regime.¹¹.² Note that the mixing enhancement was achieved without thrust loss in these experiments. Through a detailed investigation of the physics of the vortex generation mechanism, Kim and Samimy² concluded that the spanwise pressure gradient on the modified trailing-edge surfaces is the major source of streamwise vorticity.

The reason for the use of the trailing-edge modifications on the splitter plate or on the nozzle extension in a full nozzle was to simplify the problem so that the physics of the streamwise vorticity generation mechanism could be identified. However, in the practical applications, the cutouts would be located on the nozzle blocks, that is, before the expansion in the nozzle diverging section is completed. The purpose of the present experiments is to show that a cutout on the contoured nozzle block is effective in mixing enhancement in all flow regimes, including the ideally expanded regime.

Experimental Facility and Techniques

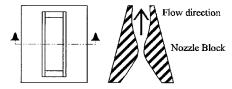
The air delivery system to the nozzle is similar to the one used by Kim and Samimy² with additional flow conditioning screens installed in the stagnation chamber.³ As in the previous experiments,² the nozzle exit measures 2.86 cm wide and 0.95 cm high, with an equivalent diameter $[D_{eq} = (4A_{exit}/\pi)^{1/2}]$ of 1.86 cm. The schematic of the baseline nozzle and the types of cutouts on the nozzle block are shown in Fig. 1. Contrary to the previous trailingedge modifications, in which the cutouts were either on the splitter plate¹ or on the nozzle extension in a full nozzle,² the cutouts are located on the contoured nozzle blocks. These contoured nozzles are designed to generate uniform flows at the nozzle exit, which operated at the design conditions. The nozzle block has a cutout of either rectangular type on the side (RS), shown in Fig. 1b, or rectangular type at the center (RC), shown in Fig. 1c. The cutouts on the nozzle block are more representative of practical applications. This is the only major difference between the present nozzles and the previous nozzles. The cutout dimensions are the same as those used in the previous experiments.^{1,2} The wall thickness at the cutout edge in the present case is gradually decreased from 4.5 mm at the beginning of the cutout to 1 mm at the nozzle exit, whereas it was 1 mm in the previous cases.

As in previous work, the instantaneous cross-sectional images were acquired by the laser sheet illumination technique. ^{1.2} The visualizations of the jet cross section were performed at four downstream locations, that is, $x/D_{\rm eq}=1,2,4$, and 8. The jet was operated at three fully expanded jet Mach numbers of 1.75, 2.0 (design Mach number), and 2.5.

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a) Side and end views of the baseline nozzle

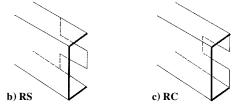
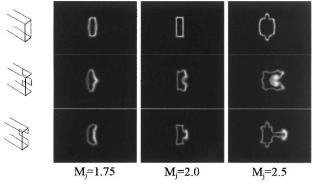


Fig. 1 Schematic of nozzle configurations.



 $a) x/D_{\rm eq} = 2$

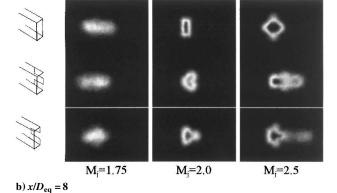


Fig. 2 Average cross-sectional images; physical dimensions of each image are a) 116.4 mm $(6.3D_{\rm eq})$ wide and 77.6 mm $(4.2D_{\rm eq})$ high and b) 178.5 mm $(9.6D_{\rm eq})$ wide and 119.0 mm $(6.4D_{\rm eq})$ high.

Results

From the average and instantaneous jet cross-sectional images at four downstream locations, the jet evolution and overall mixing were investigated. As in the previous experiments, 1.2 the pair of counterrotating streamwise vortices generated by each cutout dictate the cross-sectional development and mixing characteristics of the jet. The mixing area at a given downstream location was calculated and normalized with a reference mixing area. 1.2 The reference mixing area for each nozzle was acquired at $M_j = 2.0$ (ideally expanded) using the baseline nozzle after flow visualizations for the nozzle operated at three flow conditions were performed. The average jet cross-sectional images of 50 instantaneous frames at $x/D_{\rm eq} = 2$ and 8 are shown in Fig. 2. The image distortions due to the angle between the camera axis and the jet axis were corrected using image processing.

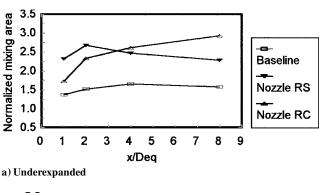
Underexpanded Case $(M_i = 2.5)$

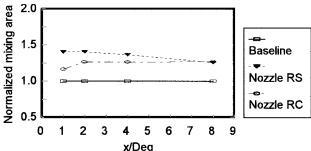
As shown in Fig. 2 and discussed in the previous experiments,² two types of counter-rotating streamwise vortices were observed

in this flow regime: a kidney type by nozzle RS and a mushroom type by nozzle RC. It is believed that the spanwise pressure gradient on the nozzle block surface around the cutout is the major source of streamwise vorticity in the underexpanded case. In the underexpanded flow regime for the base nozzle, the effects of corner vortices can be easily seen in Fig. 2 (Ref. 1).

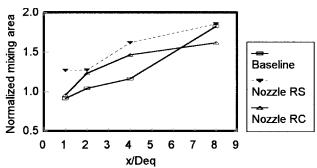
The jet cross-sectional development for nozzles with cutouts on the contoured nozzle block is quite similar to that for nozzles with cutouts on the splitter plate¹ or on the nozzle extension.² Even though M_j is the same in this and the previous experiments, the degree of underexpansion in the present case is higher. For $M_j = 2.5$, for example, the ratio of static pressure at the beginning of the cutout to the ambient pressure is 5.3, in comparison with 2.3 in the previous case. The increased underexpansion generates stronger pairs of counter-rotating streamwise vortices and in turn results in more energetic mixing. As in the previous cases, the vortex pairs interact in nozzle RS at $x/D_{\rm eq} = 2$. This interaction results in reduced growth rate after the interaction as shown in Fig. 3a. In the underexpanded flow regime, the mixing areas of nozzle RS at $x/D_{\rm eq} = 2$ and 8 show 30 and 25% increases over that of a nozzle with the same cutout located on the splitter plate.¹

For nozzle RC, a mushroom-type pair of streamwise vortices roll up farther away from the jet center when compared to that of a nozzle with the same cutout located on the splitter plate. At both $x/D_{\rm eq}=2$ and 8, the normalized mixing area of nozzle RC was increased about 20% over that of a nozzle with the same cutout located on the splitter plate. The increased spanwise pressure gradient on the contoured nozzle block with a cutout seemed to generate a stronger mushroom-type pair of counter-rotating streamwise vortices. By the pumping





b) Ideally expanded



c) Overexpanded

Fig. 3 Normalized mixing areas with downstream location.

action of this pair of vortices, a significant ejection of the jet air into the ambient air was observed at all downstream locations. Similar to previous work,² this favorable pumping action of mushroom-type vortices resulted in a continuous increase in mixing as shown in Fig. 3a. As in the previous experiments,² nozzle RS, which generates a kidney-type pair of streamwise vortices, showed better mixing in the near field, whereas nozzle RC, which generates a mushroom-type pair of streamwise vortices, showed better mixing at locations farther downstream, as shown in Fig. 3a.

Ideally Expanded Case $(M_j = 2.0)$

Significant jet cross-sectional development is observed for this flow condition by the cutout on the contoured nozzle block. With this nozzle configuration, the jet cross-sectional developments with downstream location for nozzles RS and RC are similar to those in the underexpanded flow regime of $M_j=2.2$ with similar cutouts as in previous experiments. This is expected because the ratio of the static pressure at the beginning of the cutout to the ambient pressure is 2.4, in comparison with 1.0 in the previous cases with cutouts on the splitter plate or on the nozzle extension. This underexpansion generates a surface pressure gradient around the cutout, which is a necessary condition for streamwise vorticity generation. The two pairs of counter-rotating streamwise vortices are similar to those in the $M_j=2.5\,$ case, although not strong enough to deform the jet cross section dramatically.

Mixing areas at $x/D_{\rm eq}=2$ and 8 increased about 20% when they are compared with those for nozzles with similar cutouts on the splitter plate. Nozzle RS shows better mixing in the near field as in the $M_j=2.5$ case.

Overexpanded Case $(M_i = 1.75)$

With the cutout on the contoured nozzle block, the jet cross-sectional development is significantly altered for locations far down-stream. The jet cross sections of nozzle RS and RC show an axis switching, as in the baseline nozzle, by the $x/D_{\rm eq}=8$ location for the present cases. On the other hand, only the baseline nozzle showed an axis switching by this location in the previous cases. From the instantaneous images, the degree of flapping motion of mixing layers can be inferred. In the previous cases, and the cutouts significantly reduced the flapping motion. However, the present cutout did not significantly change the flapping motion. The unaltered flapping motion is most likely responsible for the enhanced mixing.

Nozzle RS shows approximately 60% increased mixing at $x/D_{\rm eq}=8$ when compared with that of a nozzle with a similar cutout on the splitter plate, ¹ although it shows a little reduced mixing at $x/D_{\rm eq}=2$. Nozzle RC shows a reduced mixing level at $x/D_{\rm eq}=2$ and about the same mixing level at $x/D_{\rm eq}=8$ when it is compared with that of a nozzle with a similar cutout on the splitter plate. ¹ Contrary to the previous cases, the growth rates of mixing area for both nozzles RS and RC are positive all the way up to $x/D_{\rm eq}=8$. As mentioned earlier, the unaltered jet flapping motion is most likely related to the enhanced mixing.

Conclusions

A rectangular nozzle with a cutout on the contoured nozzle block showed higher mixing levels than previous experiments with the cutouts either on the splitter plate in a half nozzle or on the extension plates in a full nozzle. Except for the increased mixing level, the overall development of jet cross sections with downstream locations and the role of streamwise vortices remained similar to those of previous experiments with the cutouts on a splitter plate or a nozzle extension plate. The role of streamwise vortices in the jet development of the ideally expanded case seemed to be similar to those in the underexpanded case.

In the overexpanded flow regime, the present nozzle with a cutout on a contoured nozzle block showed about the same level of flapping motion as the baseline nozzle, whereas nozzles with cutouts on the splitter or on the extension in the previous experiments showed reduced flapping. The unaltered flapping motion of nozzles with a cutout resulted in positive growth rates all the way up to $x/D_{\rm eq}=8$, whereas the growth rates of nozzles with cutouts on nozzle extensions were negative at locations far downstream. Therefore, a nozzle

with a cutout on a contoured nozzle block would perform better in mixing at all flow conditions. Note that in the previous experiments with the cutouts either on the splitter plate in a half nozzle or on the nozzle extension in a full nozzle, there was no thrust loss associated with the cutouts. However, no thrust measurements were performed for the present cutouts.

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J. P. Gore Associate Editor

Insensitivity of Unsteady Vortex Interactions to Reynolds Number

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Introduction

S EVERAL investigations have revealed that the vortex breakdown location over delta wings exhibits quasi-periodic oscillations along the axis of the vortices due to an interaction between the two leading-edge vortices. These observations were made by flow visualizationin water-tunnelfacilities at low Reynolds numbers, and the issue arises as to whether this is a low-Reynolds-number phenomenon. In this Note, by using two-point unsteady surface pressure measurements in a wind tunnel, it is shown that this phenomenon exists at much higher Reynolds numbers.

The antisymmetric motion of breakdown locations for left and right vortices (Fig. 1) can be demonstrated by studying the difference between the breakdown locations ($X_{\rm left} - X_{\rm right}$)/c and the average breakdown location ($X_{\rm left} + X_{\rm right}$)/2c. The spectra of these are shown in Fig. 2 for $\Lambda=75$ deg and $\alpha=42$ deg (taken from Ref. 1). It is seen that most of the energy is concentrated in the difference and that there is a dominant peak corresponding to the quasi-periodic antisymmetric oscillations. Experiments on the nature and source of these oscillations as well as the effect of angle of attack and sweep angle are reported in detail in Ref. 2. Similar observations of the quasi-periodic oscillations of breakdown location were also made by others^{3–7} by using flow visualization in water tunnels. The range of Reynolds number in these water-tunnel experiments and the frequency of the organized motion are shown in Table 1. Note that these oscillations were observed at Reynolds

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